D–4468

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

First Semester

ALGEBRA — I

(CBCS–2018–2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define mappings.
2. Define subgroup. Give an example.
3. Define automorphism of a group.
4. What is meant by normal subgroup?
5. Show that the group of order 21 is not simple.
6. Define an internal direct product of groups.
7. Define a maximal ideal of a ring with an example.
8. If D is an integral domain with finite characteristic, then prove that the characteristic of D is a prime.
9. State unique factorization theorem.
10. Prove that an Euclidean ring posses an unit element.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For all $a \in G$, prove that $H_a = \{x \in G / a \equiv x \bmod H\}$.

Or

(b) Prove that HK is a subgroup of G if and only of $HK = KH$.

12. (a) Prove that, N is a normal subgroup of G if and only if $gN^{-1}g = N$ for every $g \in G$.

Or

(b) Prove that a group of order 9 is abelian.

13. (a) If υ is an ideal of R and $I \in \upsilon$, then prove that $\upsilon = \mathrm{R}$.

Or

(b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a field.

14. (a) Let R be a Euclidean ring. Suppose that for $a,b,c \in R$, $a \mid bc$ and $(a,b) = 1$. Then prove that $a \mid c$.

Or

(b) Prove that the mapping $\phi : D \to F$ defined by $\phi(a) = [\alpha, 1]$ is an isomorphism of D into F.

15. (a) State and prove Eisenstein criterion.

Or

(b) If p is a prime number of the form $4n+1$, then prove that $p = a^2 + b^2$ for some integers a and b.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$, respectively then prove that $o(HK) = \dfrac{o(H)o(K)}{o(H \cap K)}$.

17. State and prove Cauchy's theorem.

18. Prove that every finite abelian group is the direct product of cyclic groups.

19. Prove that $J[i]$ is a Euclidean ring.

20. State and prove unique factorization theorem.

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D–4469

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

First Semester

ANALYSIS – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define bounded set.

2. Define ordered field. Give an example.

3. Define metric space. Give an example.

4. Define perfect sets. Give an example.

5. State the root tests.

6. Find the radius of convergence of the power series $\sum \frac{2^n}{n^2} z^n$.

7. Define Uniformly continuous functions. Give an example.

8. Let f be defined on $[a,b]$. If f is differentiable at a point $x \in [a,b]$, then show that f is continuous at x.

9. State the intermediate theorem.

10. Define contraction.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) If x and y are complex, prove that $||x|-|y|| \leq |x - y|$.

Or

(b) Prove that the set of all rational numbers in countable.

12. (a) Show that every k-cell is compact.

Or

(b) Prove that a finite point set has no limit point.

13. (a) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Prove that $f(x)$ is compact.

Or

(b) If f is continuous mapping of a metric space X into a metric space Y and if E is connected subset of X, prove that $f(E)$ is connected.

14. (a) State and prove generalized mean value theorem.

Or

(b) State and prove Cantor intersection theorem.

15. (a) State and prove intermediate value theorem for derivatives.

Or

(b) State and prove chain rule for differentiation.

PART C — $(3 \times 10 = 30$ marks)

Answer any THREE questions.

16. For every real $x > 0$ and every integer $n > 0$, prove that there is one and only real y such that $y^n = x$.

17. Prove that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$.

18. State and prove Bolzano-Weierstrass theorem.

19. State and prove Taylor's theorem.

20. State and prove the inverse function theorem.

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D–4470

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Find all real valued solutions of the equation $y'' - y = 0$.

2. Define linearly independent and linearly dependent functions.

3. Find two linearly independent solutions of $y'' - \frac{1}{4x^2} y = 0$, $x > 0$.

4. Find the singular points of the equation $x^2 y'' + (x + x^2) y' - y = 0$ and determine whether they are regular singular points or not.

5. Write any two legendre polynomials.

6. Define indicial polynomial.

7. State the Bessel function.

8. Find the integrating factor of the function $\cos x \cos y dx - 2\sin x \sin y\, dy = 0$.

9. State the Lipschitz condition.

10. State the local existence theorem for the initial value problem $y' = f(x, y), y(x_0) = y_0$ on z.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the uniqueness theorem for initial value problem.

Or

(b) Find the solutions of the initial value problem $y'' + 10y = 0$, $y(0) = \pi$, $y' = \pi^2$.

12. (a) Obtain two linearly independent solutions of $x^2 y'' + 2x^2 y' - 2y = 0$.

Or

(b) One solution of $x^2 y'' - 2y = 0$ on $0 < x < \infty$ is $\phi_1(x) = x^2$. Find all the solutions of $x^2 y'' - 2y = 2x - 1$ on $0 < x < \infty$.

13. (a) Find all solutions of the equation $x^2 y'' + xy' - 4\pi y = x$ for $x > 0$.

Or

(b) Find the two independent solution of the equation $(3x-1)^2 y'' + (9x-3)y' - 9y = 0$ for $x > \frac{1}{3}$.

14. (a) Show that –1 and +1 are regular singular points for the legendre equation $(1-x^2)y''-2xy'+\alpha(\alpha+1)y=0$.

Or

(b) Find the first four successive approximate $\phi_0, \phi_1, \phi_2, \phi_3$ for the equation $y'=1+xy,\ y(0)=1$.

15. (a) Let f be a continuous and satisfy a Lipschitz condition on R. If ϕ and ψ are two solutions $y'=f(x, y)$, $y(x_0)=y_0$ on an interval I containing x_0, then prove that $\phi(x)=\psi(x)$ for all x in I.

Or

(b) Find the solution ϕ of $y''=1+(y')^2$ which satisfies $\phi(0)=0, \phi'(0)=0$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let ϕ_1 and ϕ_2 be two solutions of $L(y)=0$ on an interval I and let x_0 be any point of I. Then prove that ϕ_1 and ϕ_2 are linearly independent on I if and only if $W(\phi_1, \phi_2)(x_0)\neq 0$.

17. Determine the real valued solutions of

(a) $y^{(4)}-y=0$ and

(b) $y''-2iy'-y=0$.

18. Derive Bessel's function of first kind of order $\alpha, J_\alpha(x)$.

19. Find all solutions of

(a) $x^2 y'' + xy' - 4y = x$ and

(b) $x^2 y'' + xy' + 4y = 1$, $|x| > 0$.

20. Let M, N be two real valued functions which have continuous first order partial derivatives on some rectangle $R : |x - x_0| \le a$, $|y - y_0| \le b$. Prove that the equation $M(x, y) + N(x, y)y' = 0$ in exact in R if and only if $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ in R.

D–4471

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

First Semester

TOPOLOGY – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a function. Give an example.

2. State the axiom of choice.

3. Define basis for a topology.

4. State the Hausdorff space.

5. Define continuous function.

6. Is the rational Q connected? Justify your answer.

7. Define compact space. Give an example.

8. Define limit point compact. Give an example.

9. Define normal space.

10. State the Urysohn lemma.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Or

(b) Prove that the set $\mathcal{S}(Z_+)$ of all subsets of Z_+ is uncountable.

12. (a) Prove that the lower limit topology τ' on $\mathbb{R}$ is strictly finer than the standard topology τ.

Or

(b) Prove that every finite point set in a Hausdorff space is closed.

13. (a) State and prove the pasting lemma.

Or

(b) Prove that the union of a collection of connected sets that have a point in common is connected.

14. (a) State and prove the intermediate value theorem.

Or

(b) Prove that the image of a compact space under a continuous map is compact.

15. (a) Prove that every metrizable space is normal.

Or

(b) Prove that every well ordered set X is normal in the order topology.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that every non-empty subset of Z_+ has a smallest element.

17. Let $\bar{d}(a,b) = \min\{|a-b|, 1\}$ be the standard bounded metric on $\mathbb{R}$. If x and y are two points of $\mathbb{R}^w$, define $D(x,y) = l.u.b\left\{\frac{\bar{d}(x_1, y_1)}{i}\right\}$. Prove that D is a metric that induces the product topology on $\mathbb{R}^w$.

18. Prove that every compact subset of a Hausdorff space is closed.

19. Prove that the product of finitely many compact space is compact.

20. State and prove Urysohn metrization theorem.

D–4472

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Second Semester

ALGEBRA – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. When will you say that a vector space V is finite dimensional?

2. Define a basis of a vector space.

3. Define internal direct sum.

4. What is meant by the annihilator of W?

5. Define algebraic number.

6. Write short notes on the splitting field over F.

7. What is meant by the Galois group of $f(x)$?

8. Define a characteristic vector of T.

9. Prove that $T \in A(V)$ is unitary if and only if $TT^* = 1$.

10. Define Hermitian and skew-Hermitian.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) If A and B are subspaces of V, prove that $(A+B)/B$ is isomorphic to $A/(A \cap B)$.

Or

(b) Prove that $L(S)$ is a subspace of V.

12. (a) Prove that $W^{\perp}$ is a subspace of V.

Or

(b) If $u, v \in V$ then prove that $|\langle u, v\rangle| \leq \|u\| \cdot \|v\|$.

13. (a) If V is finite dimensional and W is a subspace of V, then prove that $\hat{W}\,W$ is isomorphic to $\hat{V}\,/\,A(W)$ and $\dim A(W) = \dim V - \dim W$.

Or

(b) State and prove Bessel's inequality.

14. (a) If $\alpha_1, \alpha_2, \alpha_3$ are the roots of the cubic polynomial $x^3 + 7x^2 - 8x + 3$, find the cubic polynomial whose roots are $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \frac{1}{\alpha_3}$.

Or

(b) If $T \in A(V)$ is such that $\langle vT, v\rangle = 0$ for all $v \in V$, then prove that $T = 0$.

15. (a) If N is normal and $AN = NA$, then prove that $AN^* = N^* A$.

Or

(b) Prove that the normal transformation N is

(i) Hermition if and only if its characteristic roots are real.

(ii) Unitary if and only if its characteristic roots are all of absolute value 1.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If S, T are subsets of V, then prove that

(a) $S \subset T$ implies $L(S) \subset L(T)$

(b) $L(S \cup T) = L(S) + L(T)$

(c) $L(L(S)) = L(S)$

17. If V and W are of dimensions m and n_1 respectively, over F, then prove that $Hom(V, W)$ is of dimension mn over F.

18. Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non-trivial common factor.

19. State and prove the fundamental theorem of Galoi's theory.

20. Prove that the multiplicative group of non-zero elements of a finite field is cyclic.

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D–4473

Sub. Code
31122

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Second Semester

ANALYSIS – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define the upper and lower Riemann integrals of f over $[a, b]$.

2. Define an refinement of a partition.

3. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

4. Define an equicontinuous function on a set.

5. Prove that $\lim_{h \to 0} \dfrac{E + (z + h) - E(z)}{h} = E'(z)$.

6. Define measurable set.

7. If A and B are two sets in M with $A \subset B$, then prove that $mA \le mB$.

8. Define a simple function.

9. Define Lebesgue integral.

10. Define an integrable function over the measurable set.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\int_{-a}^{b} f(dx) \leq \int_{a}^{-b} f\,d\,x$.

Or

(b) Suppose f is a bounded real on $[a, b]$ and $f^2 \in R$ on $[a, b]$. Does it follow that $f \in R$?

12. (a) State and prove integration by parts theorem.

Or

(b) State and prove Cauchy criterion for uniform convergence.

13. (a) Prove that every non-constant polynomial with complex coefficients has a complex root.

Or

(b) State and prove Parseval's theorem.

14. (a) If E_1 and E_2 are measurable, then prove that $E_1 \cup E_2$ is measurable.

Or

(b) If f is a measurable function and $f = g$, then prove that g is measurable.

15. (a) If $f \in \mathcal{L}(\mu)$ on E, then prove that $|f| \in \mathcal{L}(\mu)$ on E, and $\left|\int_E f \, d\mu\right| \leq \int_E |f| \, d\mu$.

Or

(b) State and prove Fatou's lemma.

PART C — $(3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove uniform convergence theorem.

17. State and prove the Stone-Weierstrass theorem.

18. If $x > 0$ and $y > 0$, then prove that $\int_0^1 t^{x-1}(1-t)^{y-1} dt = \frac{\Gamma(x)\Gamma y}{\Gamma(x+y)}$

19. State and prove Egoroff's theorem.

20. State and prove Lebessgue's dominated convergence theorem.

D–4474

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Second Semester

TOPOLOGY – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State the countable intersection property.

2. Define compactification in a space.

3. Define locally compact. Give an example.

4. Define a G_δ-set. Give an example.

5. Is two compactifications equivalent? Justify your answer.

6. What is meant by the point open topology?

7. Define the evaluation map.

8. Define compact convergence topology.

9. Define a Baire space. Give an example.

10. Define general position in $\mathbb{R}^N$.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let X be a Hausdorff space. Prove that X is locally compact if and only if given x in X and a neighborhood $\mho$ of x, there is a neighborhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset \mho$.

Or

(b) If X is completely regular and non-compact, then prove that $\beta(X)$ is not metrizable.

12. (a) Prove that every closed subspace of a paracompact space is paracompact.

Or

(b) Prove that the product of completely regular space is completely regular.

13. (a) Let X be completely regular. Show that X is connected if and only if $\beta(X)$ is connected.

Or

(b) Let X be normal and let A be closed G_δ-set in X. Prove that there is a continuous functions $f: X \to [0,1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) If X is locally compact or X satisfies the first countability axiom, then prove that X is completely regular.

Or

(b) Show that the sets $B_C(f, \varepsilon)$ form a basis for a topology of Y^X.

15. (a) Prove that every open subset of a Baire space is a Baire space.

Or

(b) Prove that every compact subset of $\mathbb{R}^N$ has topological dimension at most N.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove the Tychonoff theorem.

17. State and prove the stone cech compactification theorem.

18. Let $I = [0,1]$. Prove that there exists a continuous map $f : I \rightarrow I^2$ whose image fills up the entire square I^2.

19. State and prove Ascoli's theorem.

20. Let $X = Y \cup Z$, where Y and Z are closed sets in X having finite topological dimensions. Prove that $\dim X = \max\{\dim Y, \dim Z\}$.

D–4475

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define Pfaffian differential equation.

2. What is meant by orthogonal equation trajectories?

3. Define self-orthogonal.

4. Write the form of the linear PDE of order one.

5. Solve : $a(p+q) = z$.

6. What is the general solution of $Pp + Qq = R$?

7. Show that the differential equations $\frac{\partial Z}{\partial x} = x^2 - ay$ and $\frac{\partial z}{\partial y} = y^2 - ax$ are compatible.

8. Solve $\left(2D^2 - 5DD' + 2D'^2\right)z = 0$.

9. Solve : $\frac{\partial^2 z}{\partial x^2} = \frac{1}{a} - xy$.

10. Write down the interior Neumann boundary value problem.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Solve : $\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{zxy - 2x^2}$.

Or

(b) Solve : $\frac{x\,dx}{y^3 x - 2x^4} = \frac{dy}{2y^4 - x^3 y} = \frac{dz}{9z\left(x^3 - y^3\right)}$.

12. (a) Show that

$(yz + 2x)\,dx + (zx - 2z)\,dy + (xy - 2y)\,dz = 0$ is integrable.

Or

(b) Find the differential equation of the set of all right circular cones whose axes coincide with z-axis.

13. (a) Find the equation of surface satisfying $4yz\,p + q + 2y = 0$ and passing through $y^2 + z^2 = 1,\ x + z = 2$.

Or

(b) Find the family orthogonal to

$\phi\left[z\,(x + y)^2, x^2 - y^2\right] = 0$.

14. (a) Solve $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = xyz$.

Or

(b) Find the complete integral of $q = px + p^2$.

15. (a) Reduce $\frac{\partial^2 z}{\partial x^2} = (1+y)^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

Or

(b) Derive D'Alembert's solution to the one-dimensional wave equation.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that the orthogonal trajectories of the family of conics $y^2 - x^2 + 4xy - 2cx = 0$ consists of a family of cubics with the common asymptote $x + y = 0$.

17. Solve $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$.

18. Find the complete integral of $xp^2 - ypq + y^3 q - y^2 z = 0$.

19. Solve : $\left(D^2 - DD' - 2D'^2 + 2D + 2D'\right)z = e^{2x+3y} + xy$.

20. A taut string of length $2l$ is fastened at both ends. The mid-point of the string is taken to a height b and then released from reset in that position. Find the displacement of the string at time t.

D–4476

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. What is meant by binomial line?

2. Define an osculating circle.

3. What is meant by point of inflexion?

4. Define evolute.

5. Write short notes on the general helicoid.

6. Explain geodesic parallels.

7. Write the canonical equations for geodesies.

8. Define the osculating development of the curve.

9. Define principal curvature.

10. Define developable.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the equation of the osculating plane of the curve given by

$$\vec{x} = \{a \sin u + b \cos u, a \cos u + b \sin u, c \sin 2u\}.$$

Or

(b) With the usual notations, prove that $\left[\vec{\gamma}', \vec{\gamma}'', \vec{\gamma}'''\right] = k^2 \lambda$.

12. (a) For any general helix c prove the following relation between its curvature and that of the plane curve c_1 obtained by projecting c on a plane orthogonal to its axis $k = k_1{}^2 \sin^2 \alpha$.

Or

(b) Find the coefficients of direction which makes an angle $\pi/2$ with the direction whose coefficients are (l, m).

13. (a) Show that every helix on a cylinder is a geodesic.

Or

(b) If a curve lies on a sphere show that ρ and σ are related by $\frac{d}{ds}(\sigma \rho') + \frac{\rho}{\sigma} = 0$.

14. (a) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z = a$ constant.

Or

(b) Find the geodesic curvature of the parametric curve $v = c$, where c is a constant.

15. (a) Explain an Umbilic point.

Or

(b) Write a short note on “Dupins Indicatrix”.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Derive Serret–Frenet formula.

17. Show that the intrinsic equations of the curve given by $x = ae^{u} \cos u, y = ae^{u} \sin u, z = be^{u}$ are

$$k = \frac{a\sqrt{2}}{\left(2a^2 + b^2\right)^{1/2}}, \frac{1}{s}; I = \frac{b}{\left(2a^2 + b^2\right)^{1/2}}; \frac{1}{s}.$$

18. Find a surface of revolution which is isometric with a regions of the right helicoid.

19. State and prove Gauss–Bonnet theorem.

20. Prove that a necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normals along the curve form a developable.

D–4477

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define oriented arc.

2. What is meant by connected network?

3. Define critical path.

4. Define optimal feasible solution.

5. Explain bounded variables.

6. Define two person–zero sum game.

7. Explain the sensitivity analysis.

8. Define concave and convex function.

9. Define separable function – Give an example.

10. Define quadratic programming model.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain Dijkstra's algorithm to find the shortest route.

Or

(b) Construct the network diagram comprising activities $B, C, \ldots.. Q$ and N such that the following constraints are satisfied $B < E, F$; $C < G, L$; $E, G < H$; $L, H < I$; $L < M$; $H < N$, $H < J$; $I, J < P$; $P < Q$ the notation $x < y$ means that the activity x must be finished before y can begin.

12. (a) Explain EOQ with price breaks.

Or

(b) Explain the maximal flow problem.

13. (a) Solve the LPP by using simplex method.

Maximize : $z = 30x_1 + 20x_2$

Subject to

$$10x_1 + 8x_2 \leq 800$$
$$x_1 \leq 60$$
$$x_2 \leq 75$$
$$x_1, x_2 \geq 0$$

Or

(b) Solve the following game

		Player B			
		1	2	3	4
Player A	1	8	2	9	5
	2	6	5	7	18
	3	7	3	–4	10

14. (a) Solve by Jacobian method

Maximize $z = 2x_1 + 3x_2$

Subject to

$$x_1 + x_2 + x_3 = 5$$
$$x_1 - x_2 + x_9 = 3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Or

(b) Solve the non linear programming problem by using Lagcangian multipliers

Minimize $z = x_1^2 + x_2^2 + x_3^2$

Subject to $4x_1 + x_2^2 + 2x_3 = 14$.

15. (a) Use the Kuhn–Tucker conditions to solve the non linear programming problem

Maximize $z = 2x_1^2 + 12x_1 x_2 - 7x_2^2$

Subject to $2x_1 + 5x_2 \leq 98$.

Or

(b) Solve the following quadratic problem

Minimize $z = x_1^2 - 2x_1 x_2 + 2x_2^2 - 2x_1 - 5x_2$

Subject to

$$2x_1 + 3x_2 \leq 20$$
$$3x_1 - 5x_2 = 4$$
$$x_1 \geq 0, \text{ and } x_2 \geq 0$$

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Construct the dual of the problem

Minimize $z = 3x_1 + 10x_2 + 2x_3$

Subject to

$$2x_1 + 3x_2 + 2x_3 \leq 7$$
$$3x_1 - 2x_2 + 4x_3 = 3$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

17. The payoff matrix of a two person zero–sum game is :

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \text{Player A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \left|\begin{array}{ccc} 1 & 2 & 1 \\ 0 & -4 & -1 \\ 1 & 3 & -2 \end{array}\right| \end{array}$$

Determine the number of saddle points and the corresponding optimal solutions.

18. Reduce the game by dominance property and solve it

Player B

		1	2	3	4	5
Player A	I	1	3	2	7	4
	II	3	4	1	5	6
	III	6	5	7	6	5
	IV	2	0	6	3	1

19. Derive the optimal solution from the Kuhn–Tuker conditions for the problem.

Minimize : $z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

Subject to

$$x_1 + 3x_2 \leq 6$$

$$5x_1 + 2x_2 \leq 10$$

and $x_1 \geq 0, x_2 \geq 0$.

20. Solve the quadratic programming problem.

Minimize : $f(x_1, x_2) = -4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2$

Subject to

$$2x_1 + x_2 \leq 6$$

$$x_1 - 4x_2 \leq 0$$

$$x_1 \geq 0, x_2 \geq 0$$

D–4478

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State Euclid's Lemme.

2. Define prime number and composite number. Give example.

3. State the Euler totient function $\phi(n)$.

4. When will you say the functions are division functions?

5. State the little fermat theorem.

6. Determine whether -10_4 is e quadratic residue or non-residue of the prime 997.

7. Prove that $[-x]=\begin{cases} -[x] & \text{if} \quad x=[x] \\ -[x]-1 & \text{if} \quad x \neq [x] \end{cases}$.

8. State Wilson's theorem.

9. What are the quadratic residues and non residues $\bmod 13$?

10. Write the Diophantive equations.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any positive integer m, prove that

$(ma, mb) = m(a, b)$.

Or

(b) Find the values of x and y to satisfy

$243x + 198y = 9$.

12. (a) If $x \equiv y \pmod m$ then prove that $(x, m) = (y, m)$.

Or

(b) State and prove the division algorithm.

13. (a) If $x \geq 1$ then prove that $\sum_{n > x} \frac{1}{n^s} = 0(x^{1-s})$ if $s > 1$.

Or

(b) If $x \geq 1$, prove that $\sum_{n \leq x} \frac{1}{n} = \log x + c + 0\left(\frac{1}{x}\right)$.

14. (a) If a prime p does not divide a, then prove that

$a^{p-1} = 1 \pmod p$.

Or

(b) Prove that an arithmetic function f has a multiplicative inverse if and only if $f(1) \neq 0$. Also prove that, if an inverse exists it is unique.

15. (a) State and prove Gauss lemma.

Or

(b) If Q is odd and $Q > 0$, then prove that

$\left(\frac{-1}{Q}\right) = (-1)^{(Q-1)/2}$ and $\left(\frac{2}{Q}\right) = (-1)^{(Q^2-1)/8}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove the fundamental theorem of arithmetic.

17. Show that $a(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right)$.

18. Let f be multiplicative prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) = f(n)$ for all $n \geq 1$.

19. State and prove Euler's summation formula.

20. State and prove that quadratic reciprocity law.

D–4479

Sub. Code
31134

DISTANCE EDUCATION

B.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Stochastic processes.

2. When do you say that a Markov chain have countable state space?

3. Define order of a Markov chain.

4. Define diffusion processes.

5. Write down the equation of motion of a Brownian particle.

6. Define generating function.

7. What is meant by traffic intensity?

8. What is meant by rate equality principle?

9. Define busy period.

10. State the Erlang's loss formula. $B\left(s, \lambda/\mu\right)$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain random walk between two barriers.

Or

(b) Let $\{x_n, n \geq 0\}$ be a Markov chain with three states 0,1,2 and with transition matrix $\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ and the initial distribution.

$\Pr\{x_i = i\} = 1/3, i = 0,1,2.$. Find

$\Pr(x_1 = 1 | x_0 = 2), \Pr(x_2 = 2, x_1 = 1 | x_0 = 2)$ and $\Pr(x_3 = 1, x_2 = 2, x_1 = 1, x_0 = 2)$

12. (a) If $\{N(t)\}$ is a poisson process and $s < t$ then prove that $\Pr\{N(s) = K | N(t) = n\} = \binom{n}{k}\left(s/t\right)^k \left(1 - \left(s/t\right)\right)^{n-k}$.

Or

(b) Explain Birth-immigration process.

13. (a) If $x(t)$ with $x(0)$ and $\mu = 0$, is a Wiener process and $0 < s < t$, show that for at least one τ satisfying $s \leq \tau < t, \Pr\{x(\tau) = 0\} = \left(2/\pi\right)\cos^{-1}\left(\left(s/t\right)^{1/2}\right)$.

Or

(b) Show that for an irreducible ergodic process

$$\lim_{t \to \infty} \frac{M(k,j,t)}{M(h,i,t)} \to \frac{v_j}{v_i}.$$

14. (a) Explain waiting time in the queue.

Or

(b) Find the moments of the waiting time T.

15. (a) Derive Pollaczek – Khinchine formula.

Or

(b) Show that the distribution of $N(t)$ is poisson with mean $\lambda at = \lambda \int_o^t \{1 - B(u)\} du$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that the p.g.f. of a non-homogeneous process $\{N(t); t \geq 0\}$ is $Q(s,t) = \exp\{m(t)(s-1)\}$.

Where $m(t) = \int_0^1 \lambda(x) dx$ is the expectation of $N(t)$.

17. Derive the forward and backward Chapman – Kolmogorov differential equations.

18. The number of accident in a town follows a poisson process with a mean 2 per day and the number x_i of people involved in the i^{th} accident has the distribution (Independent) $\Pr\{x_i = k\} = \frac{1}{2^k}, k \geq 1$. Find the mean and the variance of the number of people involved in accidents per week.

19. Find the distribution of the number (Q) in the queue in steady state in an $M/M/1$ queue with rates λ, μ. Find $E(Q)$ and $Var(Q)$. Verify that $E(Q) = \lambda E(W_Q)$. Also find $Var(W_Q)$.

20. Derive the Erlang's second formula.

D–4480

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Fourth Semester

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define regular graph. Give an example.

2. Define a path with an example.

3. Define a cut vertex of a graph.

4. Define tree. Give an example.

5. Define clique of a graph.

6. What is meant by edge chromatic number of a graph?

7. Define a dual graph.

8. Define planar and non-planar graph.

9. Define in degree and out degree of a vertex.

10. Define directed walk.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every cubic graph has an even number of points.

Or

(b) With usual notations prove that $\alpha + \beta = p$.

12. (a) With usual notations prove that $K \leq K' \leq \delta$.

Or

(b) Define a block of a graph with an example. If G is a block with $v \geq 3$, then prove that any two edges of G lie on a common cycle.

13. (a) State and prove Berge theorem.

Or

(b) Prove that a graph G with atleast two vertices is biparticle if and only if all its cycles are of even length.

14. (a) If G is a tree with n vertices, $n \geq 2$, then prove that $f(G, \lambda) = \lambda(\lambda - 1)^{n-1}$.

Or

(b) State and prove Dirac theorem.

15. (a) Define a planar graph. Show that the complete graph K_5 is non-planar.

Or

(b) If two diagrams are isomorphic then prove that the corresponding vertices have the same degree pair.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove character theorem.

17. Prove that every uniquely n-colourable graph is $(n-1)$ connected.

18. State and prove Vizing's theorem.

19. (a) Show that $K_{3,3}$ is non-planar.

    (b) If G is a simple graph, then prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

20. Prove that a weak diagraph D is Eulerian if and only if every vertex of D has equal in degree and out degree.

D–4481

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Banach space.

2. Define Compact set.

3. Define bounded linear functional.

4. Define inner product space. Give an example.

5. Define project map.

6. Define orthogonal set.

7. Define self-adjoint operator.

8. Define closed convex hull.

9. Define convex functional.

10. State the closed graph theorem.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If (X, d) and (Y, d') are metric spaces where $f: X \rightarrow Y$ then prove that f is continuous at the point x if and only if for every sequence $\{x_n\}$ converging to x, $f(x_n) \rightarrow f(x)$.

Or

(b) Let (X, d) be complete and let A be totally bounded. Prove that A is relatively compact.

12. (a) State and prove Zorn's lemma.

Or

(b) If X is a finite dimensional linear space, then prove that all linear functionals are bounded.

13. (a) Let X be a real inner product space and let $x, y \in X$. Prove that $< x, y >= \frac{1}{4}\left[\|x+y\|^2 - \|x-y\|^2\right]$.

Or

(b) Prove that the eigen vectors associated with distinct eigen values of a self-adjoint linear transformation are orthogonal.

14. (a) If X is an inner product space, prove that the inner product $< x, y >$ is a continuous mapping $X \times X$ into F.

Or

(b) Let X be an inner product space and let $A = \{x_\alpha\}_{\alpha \in \wedge}$, be an orthonormal set in X. Prove that space if for any $x \in X$, $\|x\|^2 = \sum_\alpha |< x, x_\alpha >|^2$, then A is complete.

15. (a) If T is an operator on H for which $< Tx, x >= 0$ for all x, then prove that $T = 0$.

Or

(b) Prove that a bounded linear operator T on H is unitary if and only if it is a linear isometry of H onto itself.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that all compact sets are countably compact.

17. Let $\tilde{X}$ denote the normed linear space of all bounded linear functionals over the normed linear space X. Prove that $\tilde{X}$ is a Banach space.

18. State and prove Riesz theorem.

19. State and prove uniform boundedness theorem.

20. State and prove open mapping theorem.

D–4482

Sub. Code
31143

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define multiple root and simple root of $P_n(x) = 0$.

2. Define the order of the iteration method.

3. State the Descarte's rule of sign.

4. Define eigen vectors.

5. Write the Newton's interpolating polynomial formula.

6. State the Weierstrass approximation theorem.

7. What is meant by optimal value?

8. Write down the Euler's Back-ward formula.

9. Define grid points.

10. Explain the difference equations.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) By using Regula-Falsi method, determine the root of $\cos x - x e^{x} = 0$.

Or

(b) Perform two iterations of the Bairstow method to extract a quadratic $x^2 + px + q$ from the polynomial $p_3(x) = x^3 + x^2 - x + 2 = 0$.

12. (a) Find the values of a for which the matrix $A = \begin{bmatrix} 1 & a & a \\ -a & 1 & a \\ a & a & 1 \end{bmatrix}$ are positive definite.

Or

(b) Find the interval which contains the eigen values of the symmetric matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$.

13. (a) From the data :

x :	0	1	2	3
$f(x)$:	1	2	33	244

Fit quadratic splines with $M(0) = f''(0) = 0$ and hence find an estimate of $f(2.5)$.

Or

(b) Obtain a linear polynomial approximation to the function $f(x) = x^3$ on $[0, 1]$ using least square approximation with $w(x) = 1$.

14. (a) Consider the four point formula :

$$f'(x_2) = \frac{1}{6h}\left[-2f(x_1) - 3f(x_2) + 6f(x_3) - f(x_4)\right] +$$

$TE + R.E$ where $x_j = x_0 + jh,\ j = 1, 2, 3, 4$ and TE, RE are respectively the truncation error and round off error. Determine the form of TE and RE.

Or

(b) Use the Euler method to solve numerically the initial value problem $u' = -2tu^2,\ 4(0) = 1$ with $h = 0.1$ on $[0,1]$.

15. (a) Discretize $y'' = t + y,\ y(1) = 0$ using backward Euler method and compute $y(1.2)$ using $h = 0.1$.

Or

(b) Find the three term Taylor's series solution for the third order initial value problem $W''' + WW'' = 0$, $W(0) = 0, W'(0) = 0, W''(0) = 1$. Find the bound on the error for $t \in [0, 0.2]$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Use synthetic division and perform two interations of the Birge – Vieta method to find the smallest positive root of the polynomial $P_3(x) = 2x^3 - 5x + 1 = 0$. Use the initial approximation $P_0 = 0.5$.

17. Find all the eigen values of $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ using Jacobi method. Iterate till the off-diagonal elements, in magnitude, are less than 0.0005.

18. Obtain a least squares fit of the form $f = a e^{-3t} + b e^{-2t}$ from the following data :

t :	0.1	0.2	0.3	0.4
f :	0.76	0.58	0.44	0.35

19. Use Taylor's series method of order four to solve $u' = t^2 + u^2,\ u(0) = 1$ for the interval $(0, 0.4)$ using two subintervals of length 0.2.

20. Use the classical Runge-kutta method of fourth order to find the numerical solution at $x = 0.8$ for $\frac{dy}{dx} = \sqrt{x + y},\ y(0.4) = 0.41$. Assume the step length $h = 0.2$.

D–4483

Sub. Code
31144

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2024.

Fourth Semester

PROBABILITY AND STATISTICS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Sample Space.

2. If the sample space is $\mathcal{C} = C_1 \cup C_2$ and if $P(C_1) = 0.8$ and $P(C_2) = 0.5$, then find $P(C_1 \cap C_2)$.

3. Define distribution function of X and Y.

4. What is meant by conditional probability?

5. Let $f(x, y) = \begin{cases} 2, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

   Show that the correlation coefficient of X and Y is $\rho = \frac{1}{2}$.

6. Define gamma distribution.

7. The moment generating function of the random variable X is $\left(\frac{1}{3} + \frac{2}{3}e^{t}\right)^5$, find $P_r(X = 2 \text{ or } 3)$.

8. Define the concept of "Convergence in distribution".

9. When we say that a sequence of random variables converges in distribution to a random variable with distribution function?

10. State the central limit theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let $f(x)=\begin{cases} 1/x^2, & 1<x<\infty \\ 0, & \text{elsewhere} \end{cases}$, be the p.d.f. of X where $A_1=\{x:1<x<2\}$, $A_2=\{x:4<x<5\}$, find $P(A_1 \cup A_2)$ and $A(A_1 \cap A_2)$.

Or

(b) Let X and Y have the joint p.d.f.

$$f(x,\,y)=\begin{cases} 2e^{-x-y}, & 0<x<y,\, 0<y<\infty \\ 0, & \text{elsewhere} \end{cases}$$

Show that X and Y are independent.

12. (a) Determine the constant e so that

$$f(x)=\begin{cases} ex(3-x)^4, & 0<x<3 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function.

Or

(b) Prove that :

(i) $E[E(X_2/X_1)]=E(X_2)$ and

(ii) $Var\,[E(X_2/X_1)] \leq Var(X_2)$.

13. (a) Derive the p.d.f of Binomial distribution.

Or

(b) Suppose that X has a Poisson distribution with $\mu = 2$. Show that the p.d.f. of X is

$$f(x)=\begin{cases} \dfrac{2^x e^{-1}}{x!}, & x = 0, 1, 2, ... \\ 0, & \text{elsewhere} \end{cases}$$

Find $\Pr(1 \leq x)$.

14. (a) If $E(x)=17$ and $E(x^2)=298$, use Chebyshev's inequality to determine.

(i) A lower bound for $\Pr(10 < x < 24)$

(ii) An upper bound for $\Pr(|x-17| \geq 16)$.

Or

(b) One of the tasks performed by a computer operator is that of testing tapes in order to detect bad records : Ten observations in X are 67 7 35 78 28 74 5 9 37.

(i) Find the order statistics

(ii) Find the medium and 80th percentile of the sample.

15. (a) Derive the p.d.f. of chi-square distribution.

Or

(b) Let $\overline{X}_n$ denote the mean of a random sample of size n from a distribution that is $N(\mu_1, \sigma^2)$. Find the limiting distribution of $\overline{X}_n$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Derive Chebyshev's inequality.

17. Let the random variables X and Y have the joint p.d.f.

$$f(x, y) = \begin{cases} x + y, & 0 < x < 1, \quad 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Compute the correlation coefficient of X and Y.

18. Find the moment generating function, mean and variance of the gamma distribution.

19. Let $T = \frac{W}{\sqrt{V/\gamma}}$ where W and V are respectively normal with mean 0 and variance 1, chi = square with γ, show that T^2 has an F-distribution with parameters $\gamma_1 = 1$ and $\gamma_2 = v$.

20. State and prove central limit theorem.